WEEKLY TEST TYJ TEST 16 UNISION SOLUTION Date 08-12-2019

[PHYSICS]

1. Let displacement equation of particle executing SHM is

$$y = a \sin\omega t$$

As particle travels half of the amplitude from the equilibrium position, so

$$y = \frac{a}{2}$$

Therefore, $$\frac{a}{2} = a \sin\omega t$$

or $$\sin\omega t = \frac{1}{2} = \sin\frac{\pi}{6}$$

or $$\omega t = \frac{\pi}{6}$$

or $$t = \frac{\pi}{6\omega}$$

or

or $$t = \frac{\pi}{6\left(\frac{2\pi}{T}\right)} \qquad \left(\text{as } \omega = \frac{2\pi}{T}\right)$$

60. Given,

and

or $$t = \frac{T}{12}$$

Hence, the particle travels half of the amplitude from equilibrium in $\frac{T}{12}$ s.

2.
3.

4. Time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

It is independant of the mass of the bob. Therefore time period of the pendulum will remain T.

5.

6. **(b)** Use the law of conservation of energy. Let x be the extension in the spring.

Applying conservation of energy

$$mgx - \frac{1}{2}kx^2 = 0 - 0 \Rightarrow x = \frac{2mg}{k}$$

7. **(c)** The two displacement equations are $y_1 = a \sin(\omega t)$

and $y_2 = b \cos(\omega t) = b \sin\left(\omega t + \frac{\pi}{2}\right)$

$$y_{eq} = y_1 + y_2$$
$$= a \sin \omega t + b \cos \omega t$$
$$= a \sin \omega t + b \sin\left(\omega t + \frac{\pi}{2}\right)$$

Since the frequencies for both SHMs are same, resultant motion will be SHM.

Now $A_{eq} = \sqrt{a^2 + b^2 + 2ab \cos\frac{\pi}{2}}$

8. **(d)** As springs are connected in series, effective force constant

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

9. **(c)** $n = \frac{1}{2\pi}\sqrt{\frac{K_{effective}}{m}}$

Springs are connected in parallel

$$K_{eff} = K_1 + K_2 = K + 2K = 3K$$

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{(K + 2K)}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$$

10. **(a)** As springs are connected in series, effective force constant

$$\frac{1}{k_{eff}} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \Rightarrow k_{eff} = \frac{k}{2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2M}}$$

11. The motion of planets around the sun is periodic but not simple harmonic motion.

12. For freely falling case the effective g is zero, so that frequency of oscillation will be zero.

As $f = \frac{1}{2\pi}\sqrt{\frac{g_{eff}}{\lambda}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{0}{\lambda}}$$

$$f = 0$$

13. $x(t) = A\cos(\omega t + \phi)$
where, ϕ is the phase constant.

14. $$y_1 = 5[\sin 2\pi t + \sqrt{3}\cos 2\pi t]$$
$$= 10\left[\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right]$$
$$= 10\left[\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t\right]$$
$$= 10\left[\sin\left(2\pi t + \frac{\pi}{2}\right)\right] \Rightarrow \quad A_1 = 10$$

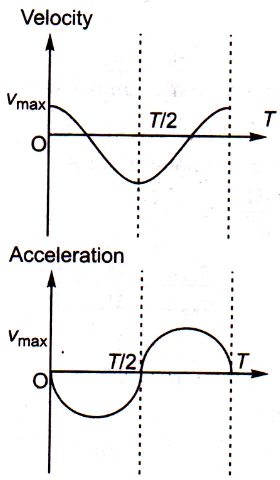
Similarly, $y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

$\Rightarrow$ $A_2 = 5$

Hence, $\frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$

15. Phase difference $\Delta\phi = \phi_1 - \phi_2$
$$= \frac{3\pi}{6} - \frac{\pi}{6}$$
$$= \frac{2\pi}{6} = \frac{\pi}{3}$$

16. In SHM, the acceleration is ahead of velocity by a phase angle $\frac{\pi}{2}$.



$$v = A\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$
$$a = A\omega^2\sin(\omega t + \pi)$$

17. The average acceleration of a particle performing SHM over one complete oscillation is zero.

18. By using $k \propto \frac{1}{l}$
Since, one-fourth length is cut away so remaining length is $\frac{3}{4}$th, hence k becomes $\frac{4}{3}$ times *ie*, $k' = \frac{4}{3}k$.

19. For the given figure,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} \quad \ldots(i)$$

$$= \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

If one spring is removed, then $k_{eq} = k$ and

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{f}{f'} = \sqrt{2}$$

$$f' = \frac{1}{\sqrt{2}} f$$

20. In a complete cycle of SHM, potential energy varies for half the cycle and kinetic energy varies for the other half of the cycle. Thus, for a time period T, the potential energy varies for $\frac{T}{2}$ time.

21. 3

22. 5 Resultant amplitude $= \sqrt{3^2 + 4^2} = 5$

23. 2

24. 2 If t is the time taken by pendulums to come in same phase again first time after $t = 0$.

and $N_S =$ Number of oscillations made by shorter length pendulum with time period T_S.

$N_L =$ Number of oscillations made by longer length pendulum with time period T_L.

Then $t = N_S T_S = N_L T_L$

$$\Rightarrow N_S 2\pi\sqrt{\frac{5}{g}} = N_L \times 2\pi\sqrt{\frac{20}{g}} \qquad (\because T = 2\pi\sqrt{\frac{l}{g}})$$

$$\Rightarrow N_S = 2N_L \text{ i.e. if } N_L = 1 \Rightarrow N_S = 2$$

25. From the relation of restitution $\frac{h_n}{h_0} = e^{2n}$ and

$$h_n = h_0(1 - \cos 60°) \Rightarrow \frac{h_n}{h_0} = 1 - \cos 60° = \left(\frac{2}{\sqrt{5}}\right)^{2n}$$

$$\Rightarrow 1 - \frac{1}{2} = \left(\frac{4}{5}\right)^n \Rightarrow \frac{1}{2} = \left(\frac{4}{5}\right)^n$$

Taking log of both sides we get

$$\log 1 - \log 2 = n(\log 4 - \log 5)$$

$$0 - 0.3010 = n(0.6020 - 0.6990)$$

$$-0.3010 = -n \times 0.097 \Rightarrow n = \frac{0.3010}{0.097} = 3.1 \approx 3$$

[CHEMISTRY]

26.

The conjugate acid-base pairs are (HCl, Cl^-) and ($CH_3COOH_2^+$, CH_3COOH).

27.

The conjugate acids are H_2O, NH_3, $HC \equiv CH$ and CH_3CH_3. Their order of acid strength is $CH_3CH_3 < NH_3 < HC \equiv CH < H_2O$. Their conjugate base follows the reverse order.

28.

NH_3 donates pair of electrons while BF_3, Cu^{2+} and $AlCl_3$ accept lone pair of electrons.

29.

$$\text{Acid} \xrightarrow{-H^+} \text{Conjugate base}, \quad \text{Base} \xrightarrow{+H^+} \text{Conjugate acid}$$

30.

H_3O^+ (acid), H_2O (conjugate base) and not OH^-.

31.

32.

For each weak polyprotic acid $K_{a_1} > K_{a_2} > K_{a_3}$

33.

H_2O is the weaker base, hence, its conjugate acid is the stronger acid,

$$H_2O + H^+ \rightleftharpoons H_3O^+$$

H_2O is the weakest acid, hence, its conjugate base is the strongest base.

$$H_2O \rightleftharpoons OH^- + H^+$$

34.

$$\text{pH [HCl]} = 2.0$$

$$\therefore \quad [H^+] = 10^{-2}\text{ M}$$

$$[HCl] = 10^{-2}\text{ M}$$

Volume = 200 mL

$$\text{pH [NaOH]} = 12.0$$

$$\text{pOH} = 2.0$$

$$[OH^-] = 10^{-2}\text{ M}$$

$$[NaOH] = 10^{-2}\text{ M}$$

Volume = 300 mL

$$N_1V_1 \text{ (acid)} = 200 \times 10^{-2} = 2$$

$$N_1V_2 \text{ (base)} = 300 \times 10^{-2} = 3$$

$$N_2V_2 > N_1V_1$$

Thus, resultant mixture basic.

$$N(OH^-) = \frac{N_2V_2 - N_1V_1}{V_1 + V_2} = \frac{3-2}{500} = 2 \times 10^{-3}\text{ M}$$

$$\text{pOH} = -\log(2 \times 10^{-3}) = 2.7$$

$$\therefore \quad \text{pH} = 14 - \text{pOH } 14 - 2.7 = 11.3$$

35.

$$[H^+] \text{ after mixing} = \frac{10^{-2} \times 10 + 10^{-4} \times 990}{1000} = \frac{0.1 + 0.0990}{1000}$$

$$= \frac{0.1990}{1000} = 1.99 \times 10^{-4}$$

$$\text{pH} = (\log 1.99 \times 10^{-4})$$

$$\therefore \quad \text{pH} = 4 - 0.3 = 3.7$$

36.

$$[H^+] = \frac{50\times10^{-1}+50\times10^{-2}}{100} = 5.5\times10^{-2}\,M$$

$pH = \log(1.99 \times 10^{-4})$

$\therefore$ $pH = 2 - 0.74 = 1.26$

37.

On heating pure water the value of ionic product of water increases i.e., $K_w = 10^{-14}$ at 25°C and at 100°C, $K_w = 10^{-12}$. Thus pH and pOH both become 6 at 100°C (pH and pOH = 7 at 25°C).

38.

(a) At 25°C, $[H^+]$ in a solution of 10^{-8} M HCl > 10^{-7} M.

(b) $[H^+] = 10^{-8}$ M.

(c) $[OH^-] = 4 \times 10^{-6}$ M $\Rightarrow$ $[H^+] = 2.5 \times 10^{-9}$ M

(d) $[H^+] = 10^{-9}$ M

39.

K_w changes with temperature. As temperature increases, $[OH^-]$ and $[H^+]$ decrease.

40.

The dissociation of H_3BO_3 is

$$H_3BO_3 + H_2O \rightarrow H_2BO_3^- + H_3O^+$$

$$K_1 = \frac{[H_2BO_3^-][H_3O^+]}{[H_3BO_3]} = \frac{(0.18).x}{(01.0)} = 7.3 \times 10^{-10}$$

or $x = [H_3O^+] = 4.1 \times 10^{-10}$

or $pH = -\log x = -\log(4.1 \times 10^{-10}) = 9.39$

41.

(a) HCl | NaOH

No. of milli eq. = $\frac{1}{10}\times100=10$ | $\frac{1}{10}\times100=10$

So solution is neutral

(b) $\frac{1}{10}\times55=5.5$ | $\frac{1}{10}\times45=4.5$

$[H^+] = \frac{1}{100} = 10^{-2}M,\ pH=2$

(c) $\frac{1}{10}\times10=1$ | $\frac{1}{10}\times90=9$ Basic

(d) $\frac{1}{5}\times75=15$ | $\frac{1}{5}\times25=5$

$[H^+] = 0.1$ M, pH = 1

42.

Initial	Final
pH = 12	pH = 11
$[H^+] = 10^{-12}$ M	$[H^+] = 10^{-11}$ M
$[OH^-] = 10^{-2}$ M	$[OH^-] = 10^{-3}$ M

Initial no. of mole of $OH^- = 10^{-2}$

Final no. of mole of $OH^- = 10^{-3}$

So no. of mole of OH^- removed = [0.1 – 0.001] = 0.009

43.

$pK_w = -\log K_w = -\log 1 \times 10^{-12} = 12.$

$K_w = [H^+][OH^-] = 10^{-12}$

$[H^+] = [OH^-]$

$\Rightarrow$ $[H^+]^2 = 10^{-12}$; $[H^+] = 10^{-6}$; $pH = -\log [H^+] = -\log 10^{-6} = 6.$

H_2O is neutral because $[H^+] = [OH^-]$ at 373 K even when pH = 6.

(d) is not correct at 373 K. Water cannot become acidic.

44.

Relative strength of weak acids = $\sqrt{\left(\frac{K_{a_1}}{K_{a_2}} \times \frac{C_1}{C_2}\right)}$

$\therefore$ Relative strength = $\sqrt{\left(\frac{K_{a_1}}{K_{a_2}}\right)}$ $(\because C_1 = C_2) = \sqrt{\left(\frac{2\times10^{-4}}{2\times10^{-5}}\right)}$

Relative strength for HCOOH to $CH_3COOH = \sqrt{10}:1$

45.

$pH = 13$

$\therefore$ $[H^+] = 10^{-13}$ M

$[OH^-] = 10^{-1}$ M = 0.1 mol L^{-1}

$[Ba(OH)_2] = 0.1$ N,

= 0.1 × 100 = 10 milliequivalents

46.

Meq. of HCl = $10 \times 10^{-1} = 1$

Meq. of NaOH = $10 \times 10^{-1} = 1$

Thus both are neutralised and 1 Meq. of NaCl (a salt of strong acid and strong base) which does not hydrolyse and thus pH = 7.

47. 7

48. $[H^+] = [OH^-]$

$K_w = [H^+]\,[OH^-] = 10^{-14}$

$\therefore [H^+] = 10^{-7}$, $pH = -\log[H^+] = 7$.

49. $pH = -\log\ [H^+]\,;[H^+] = 0.01N$

$pH = -\log\ [10^{-2}]\,;pH = 2$

50. $NaOH$ is a base, so that its $pH > 7$

[MATHEMATICS]

51. (a) Radius of circle is $\left|\frac{2+3-4}{\sqrt{5}}\right| = \frac{1}{\sqrt{5}}$

Therefore, equation is $(x-1)^2 + (y+3)^2 = \frac{1}{5}$

or $x^2 + y^2 - 2x + 6y + 1 + 9 = \frac{1}{5}$

or $5x^2 + 5y^2 - 10x + 30y + 49 = 0$.

52. (b) Centre of circle = Point of intersection of diameters = (1, –1)

Now area $= 154 \Rightarrow \pi r^2 = 154 \Rightarrow r = 7$

Hence the equation of required circle is

$(x-1)^2 + (y+1)^2 = 7^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47$.

53. (b) Centre (1, 2) and since circle touches x-axis, therefore, radius is equal to 2.
Hence the equation is $(x-1)^2+(y-2)^2=2^2$
$\Rightarrow x^2+y^2-2x-4y+1=0$.
Trick : The only circle is $x^2+y^2-2x-4y+1=0$, whose centre is (1, 2).

54. (c) $2\sqrt{g^2-c}=2a$ ....(i)
$2\sqrt{f^2-c}=2b$ ....(ii)
On squaring (i) and (ii) and then subtracting (ii) from (i), we get $g^2-f^2=a^2-b^2$.
Hence the locus is $x^2-y^2=a^2-b^2$.

55. (b) The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x-4y+4=0$ and $3x-4y-\frac{7}{2}=0$ and so it is equal to $\frac{4}{\sqrt{9+16}}+\frac{7/2}{\sqrt{9+16}}=\frac{3}{2}$. Hence radius is $\frac{3}{4}$.

56. (c) Centre is (2, 3). One end is (3, 4).
P_2 divides the join of P_1 and O in ratio of 2 : 1.
Hence P_2 is $\left(\frac{4-3}{2-1}, \frac{6-4}{2-1}\right)\equiv(1, 2)$.

57. (c) In the equation of circle, there is no term containing xy and coefficient of x^2 and y^2 are equal. Therefore $2-q=0\Rightarrow q=2$ and $p=3$.

58. (c) Here $2\sqrt{g^2-c}=2a\Rightarrow g^2-a^2-c=0$ .....(i)
and it passes through (0, b), therefore
$b^2+2fb+c=0$ ....(ii)
On adding (i) and (ii), we get $g^2+2fb=a^2-b^2$
Hence locus is $x^2+2by=a^2-b^2$.

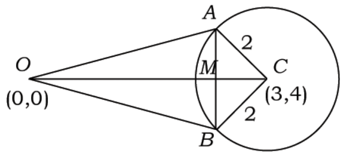
59. (d) See condition for circle and also condition for circle to pass through origin *i.e.* origin satisfies equation of circle or $c=0$.

60. (b) $x=2+3\cos\theta, y=3\sin\theta-1$
$x^2+y^2=4+9\cos^2\theta+12\cos\theta+9\sin^2\theta+1-6\sin\theta$
$=14+12\cos\theta-6\sin\theta$
$=4(2+3\cos\theta)-2(3\sin\theta-1)+4$
$\Rightarrow x^2+y^2=4x-2y+4$
$\Rightarrow (x^2-4x+4)+(y^2+2y+1)=9$
$\Rightarrow (x-2)^2+(y+1)^2=9$, $\therefore$ centre is $(2,-1)$.

61. (b) Here the equation of AB (chord of contact) is

$0+0-3(x+0)-4(y+0)+21=0$

$\Rightarrow 3x+4y-21=0$ ....(i)



CM = perpendicular distance from (3, 4) to line (i) is

$\frac{3\times3+4\times4-21}{\sqrt{9+16}}=\frac{4}{5}$

$AM=\sqrt{AC^2-CM^2}=\sqrt{4-\frac{16}{25}}=\frac{2}{5}\sqrt{21}$

$\therefore\ AB=2AM=\frac{4}{5}\sqrt{21}$.

62. (b) Point is inside, outside or on the circle as S_1 is <, >, = 0. For point (– 2, 1), $S_1<0$.

63. (c) $y=mx+c$ is a tangent, if $c=\pm a\sqrt{1+m^2}$, where $m=\tan 45^o=1$

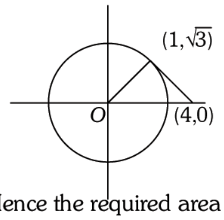
$\therefore$ The equation is $y=x\pm6\sqrt{2}$.

64. (c) Radius of given circle > Perpendicular distance from the centre of circle to the given line.

$\Rightarrow \sqrt{4+16+5}>\frac{3(2)-4(4)-m}{\sqrt{9+16}}$

$\Rightarrow \pm25>-10-m\Rightarrow m+10>\pm25\Rightarrow -35<m<15$.

65. (a) $T\equiv x+\sqrt{3}y-4=0$



Hence the required area $=\frac{1}{2}\times4\times\sqrt{3}=2\sqrt{3}$.

66. (b) Length of tangent is $\sqrt{S_1}$.

Equation of circle is $x^2+y^2-\frac{r^2}{a}=0$

Hence $S_1=\alpha^2+\beta^2-\frac{r^2}{a}$.

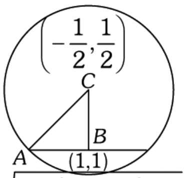
67. (c) Let $P(x_1, y_1)$ be a point on $x^2+y^2=4$. Then the equation of the tangent at P is $xx_1+yy_1=4$ which meets the coordinate axes at $A\left(\frac{4}{x_1},0\right)$ and $B\left(0,\frac{4}{y_1}\right)$. Obviously, (a) and (b) are not true.

Let (h, k) be the mid-point of AB.

Therefore $h=\frac{2}{x_1}, k=\frac{2}{y_1}$ i.e., $x_1=\frac{2}{h}, y_1=\frac{2}{k}$

But (x_1, y_1) lies on $x^2+y^2=4$.

68. (d) Here radius of circle is



$AC = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1} = \sqrt{\frac{3}{2}}$

and $CB = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{10}{4}} = \sqrt{\frac{5}{2}}$

Now $AB^2 = AC^2 - BC^2 = \frac{3}{2} - \frac{5}{2} = -1$

$\Rightarrow$ There is no possible value for AB.

Aliter : Since the point (1, 1) lies outside the circle, therefore no such chord exist.

69. (c) The common chord of given circles is

$2x - 6y - 4 - a = 0$ .....(i)

Since, $x^2 + y^2 = 4$ bisects the circumference of the circle $x^2 + y^2 - 2x + 6y + a = 0$, therefore, (i) passes through the centre of second circle *i.e.,* (1, – 3).

$\therefore 2 + 18 - 4 - a = 0 \Rightarrow a = 16$.

70. (a) It is $xx_1 + yy_1 = a^2$ *i.e.,* $5x - 3y = 10$.

71. Here the centre of circle $(3, -1)$ must lie on the line $x + 2by + 7 = 0$.

Therefore, $3 - 2b + 7 = 0 \Rightarrow b = 5$.

72. (c) Centres of circles are $C_1(2, 3)$ and $C_2(-3, -9)$ and their radii are $r_1 = 5$ and $r_2 = 8$.

Obviously $r_1 + r_2 = C_1C_2$ *i.e.,* circles touch each other externally. Hence there are three common tangents.

73. The centre of the given circle is (1, 3) and radius is 2. So, AB is a diameter of the given circle has its mid point as (1, 3). The radius of the required circle is 3.

74. Let $P = (x_1, y_1)$. The tangent at P is $xx_1 + yy_1 + 3(x + x_1) + 3(y + y_1) - 2 = 0$ .....(i)

Co-ordinates of Q satisfy (i), $5x - 2y + 6 = 0, x = 0$.

So, $3x_1 + 6y_1 + 7 = 0$ and $Q = (0,3)$

$\therefore PQ^2 = x_1^2 + (y_1 - 3)^2 = x_1^2 + y_1^2 - 6y_1 + 9$

$= 11 - 6x_1 - 12y_1$, $(\because x_1^2 + y_1^2 + 6x_1 + 6y_1 - 2 = 0)$

$= 11 - 2(3x_1 + 6y_1) = 11 - 2(-7) = 25$. So, PQ =5.

75 Radius = perpendicular distance from (1, – 3) to

$3x - 4y - 5 = 0$, *i.e.* $\left|\frac{3 + 12 - 5}{\sqrt{5^2}}\right| = 2$.